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Adaptive Wind Tunnels with Imperfect Control

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An earlier study[5] of the convergence of the adaptive-wall wind-tunnel scheme for a sinusoidal model in a two-dimensional tunnel is extended to the case where ideal matching at the interface cannot be achieved. It is assumed that, in place of the desired sinusoidal correction, a correction including an extraneous harmonic always occurs. Two different assumptions are made regarding the fitting of this distorted sinusoid to the observed error signal. It is found that the iteration converges for the same range of relaxation constants as for the ideal case, but that unconfined flow is not achieved. For reasonable numerical values, the iteration nevertheless appears to make substantial improvement in a flow involving boundary interference.

Nomenclature

a	= coefficient in Eq. (17) and Table 1
a'_n	$= a_n e^{2\beta\lambda h}$
A_n, a_n	= coefficients defined in Eq. (2), at nth iteration
$\boldsymbol{A}_n$	= the column matrix $\begin{pmatrix} A_n \\ a_n \end{pmatrix}$
b	= coefficient in Eq. (17) and Table 1
B_n, b_n,	= coefficients defined in Eq. (2), at nth iteration
C_n, c_n	= coefficients defined in Eq. (7), at nth iteration
$f(x), f^{(n)}(x)$	= wall-correction functions [Eq. (1)]
h	= value of y at the tunnel interface
$\boldsymbol{I}$	= the unit matrix
k	= "relaxation coefficient" used in the iteration
K_n	= the value, at the nth iteration, of the constant in Eq. (1)
$\boldsymbol{K}$	= modal matrix formed by the eigenvectors of $\boldsymbol{M}$
$\boldsymbol{K}^{-1}$	= inverse of $\boldsymbol{K}$
l_1, l_2	= eigenvalues of $\boldsymbol{M}$
$\boldsymbol{L}$	= diagonal matrix $\begin{pmatrix} l_1 & 0 \\ 0 & l_2 \end{pmatrix}$
$\boldsymbol{M}$	= the matrix $\begin{bmatrix} 1-\dfrac{a}{\lambda} & -\dfrac{b}{\lambda} \\ -\dfrac{\mu a}{3\lambda} & 1-\dfrac{\mu b}{3\lambda} \end{bmatrix}$
n	= the number of the iteration
V	= stream speed
$w_1^{(i)}, w_2^{(i)}$	= ith modal coefficients [Eq. (33)]
β	= Prandtl-Glauert factor $(1-V^2/a_\infty^2)^{1/2}$, where a_∞ is the speed of sound
$\delta^{(n)}u(x,h)$	= the difference between $\varphi_x^{(n)}(x,h)$ and $\phi_x^{(n)}(x,h)$ [Eq. (9)]
$\Delta A_n, \Delta a_n, \Delta B_n, \Delta\phi$	= increments of A_n, a_n, B_n, and ϕ introduced at the nth iteration
ϵ	= model amplitude Eq. (3), $\epsilon\lambda \ll 1$
λ	= model wavenumber; model wavelength $= 2\pi/\lambda$
μ	= coefficient of the extraneous harmonic [Eq. (1)]
$\phi^{(n)}(x,y)$	= perturbation potential of flow in wind tunnel at nth iteration
$\varphi^{(n)}(x,y)$	= perturbation potential of the (calculated) flowfield outside the tunnel at nth iteration

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Introduction

THE concept of the adaptive-wall wind tunnel has been presented in a number of papers.[1-3] Briefly, the proposal is to match, by successive iterations, the flowfield within a wind tunnel in the presence of an arbitrary model and test configuration, to a computed exterior flowfield that satisfies the required far-field boundary conditions.

Some studies of the convergence of this iterative process have been carried out. The most convincing of these studies are done by numerical simulation of the whole process.[3] There are also purely analytical studies, which are limited to rather simple classes of models and tunnels.[4,5] All of these studies, to date, assume ideal control of conditions at the interface between interior and exterior regions. That is to say, at any nth iteration, when an error function, say $\delta^{(n)}u(x)$, has been found, the operator is able to introduce a corresponding change, say $k\delta^{(n)}u(x)$, into the flowfield at the interface. But in a real tunnel such a correction function can only be achieved imperfectly because of nonzero size of control elements, finite number of control elements, limited number of instrument readings, etc.

We anticipate that these imperfections may have a deleterious effect on the process of convergence to unconfined flow conditions. This could (and should) be studied by numerical simulation. It is also interesting to restudy, analytically, the simple cases previously studied by Lo and Kraft[4] and by Sears.[5] We consider first the simplest, which is the sinusoidal model in a two-dimensional tunnel. In the earlier studies it was assumed that a perfect sinusoidal correction $k\delta^{(n)}u(x)$ could be introduced, and convergence was found over a range of the relaxation constant k. We now want to assume, instead, that only a distorted sinusoid can be introduced at the interface.

Effect of an Extraneous Harmonic

Suppose, for example, that when a sinusoidal correction proportional to $\sin\lambda x$ is called for, where x is the streamwise coordinate, the response can be at best a periodic function of the correct wavelength but distorted by an extraneous higher harmonic, described by

$$f(x) = \text{constant} \times (\sin\lambda x + \mu \sin 3\lambda x) \tag{1}$$

where μ is a constant not under control of the operator but determined by the design of the tunnel.

B_n, a_n, b_n replaced by

$$\left.\begin{aligned} A_{n+1} &= A_n + \Delta A_n \\ B_{n+1} &= B_n + \Delta B_n = B_n + \Delta A_n \\ &= A_n - \beta^{-1}\epsilon V + \Delta A_n = A_{n+1} - \beta^{-1}\epsilon V \\ a_{n+1} &= b_{n+1} = a_n + \Delta a_n \end{aligned}\right\} \tag{22}$$

Convergence

Combining Eqs. (20-22), we have two simultaneous linear equations for A_{n+1} and a_{n+1} in terms of A_n and a_n. Before working out the convergence of these simple recurrence formulas, we propose to simplify them by observing that for cases of practical interest $\beta\lambda h$ is probably large enough so that the negative exponentials in Eqs. (20) and (21) can be neglected; viz., $\beta\lambda h$ is π times the ratio of βh to the half-wavelength. Since the half-wavelength might be identified with "the model's chord length" and h is the tunnel half-height, it appears that $\beta\lambda h$ should be 1.0 or greater, for practical interest. If so, $e^{-\beta\lambda h}$ is small compared to $e^{\beta\lambda h}$, and $e^{-3\beta\lambda h}$ even smaller compared to $e^{3\beta\lambda h}$. We therefore neglect these smaller terms and then find it convenient to define a new coefficient

$$a'_n \equiv a_n e^{2\beta\lambda h} \tag{23}$$

The recurrence formulas Eqs. (20-22) then become

$$\begin{aligned} A_{n+1} &= \left(1-\frac{a}{\lambda}\right)A_n - \frac{b}{\lambda}a'_n \\ a_{n+1} &= -\frac{\mu a}{3\lambda}A_n + \left(1-\frac{\mu b}{3\lambda}\right)a'_n \end{aligned} \tag{24}$$

In matrix notation,

$$A_{n+1} = MA_n \tag{25}$$

where A_n denotes $\begin{pmatrix} A_n \\ a'_n \end{pmatrix}$ and M denotes the square matrix

$$\begin{bmatrix} 1-\dfrac{a}{\lambda} & -\dfrac{b}{\lambda} \\ -\dfrac{\mu a}{3\lambda} & 1-\dfrac{\mu b}{3\lambda} \end{bmatrix}$$

Equation (25) can also be written as

$$A_{n+1} = M^2 A_{n-1} = M^3 A_{n-2} = \ldots = M^n A_1 \tag{26}$$

The iterative procedure consists of starting with a wall configuration that produces a flow described by A_1 and proceeding to A_2, A_3, etc. If the process converges, we arrive at a flow described by A_n ($n\to\infty$). This limit can be studied by applying matrix theory[6] to our little matrix M.

We first find the eigenvalues of M, viz., the roots l_1, l_2 of

$$|lI - M| = \begin{vmatrix} l-1+\dfrac{a}{\lambda} & \dfrac{b}{\lambda} \\ \dfrac{\mu a}{3\lambda} & l-1+\dfrac{\mu b}{3\lambda} \end{vmatrix} = 0 \tag{27}$$

These are easily found to be

$$l_1, l_2 = 1, \quad 1-\left(\frac{a}{\lambda}+\frac{\mu b}{3\lambda}\right) \tag{28}$$

so that the diagonal matrix of eigenvalues, say L, is

$$L \equiv \begin{bmatrix} l_1 & 0 \\ 0 & l_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1-\left(\dfrac{a}{\lambda}+\dfrac{\mu b}{3\lambda}\right) \end{bmatrix} \tag{29}$$

Convergence requires $|L|<1$; viz.,

$$0 < \left(\frac{a}{\lambda}+\frac{\mu b}{3\lambda}\right) < 2 \tag{30}$$

This criterion is evaluated for our two fitting assumptions in Table 1. We see that, for both kinds of fitting, the criterion for convergence of the iteration is the same as was found[5] for ideal wall control, provided that the approximation $e^{-\beta\lambda h} \ll e^{\beta\lambda h}$ is made.

The Flowfield after Convergence

It is now interesting to determine what flow pattern is obtained in the limit. Is unconfined flow achieved in spite of imperfect wall control?

The final flow is described by Eq. (26) with $n\to\infty$. We require, therefore, the matrix M^n in this limit. An expression for M^n is given in matrix theory:

$$M^n = KL^m K^{-1} \tag{31}$$

where L is the diagonal matrix of M's eigenvalues [Eq. (29)], and K is the modal matrix formed by the eigenvectors, viz., the modes w_1/w_2 of the equation

$$(l_i I - M)\, w^{(i)} = 0 \tag{32}$$

In our problem, taking l_1, l_2 from Eqs. (28), Eq. (32) is

$$\left.\begin{aligned} &\frac{a}{\lambda}w_1^{(1)} + \frac{b}{\lambda}w_2^{(1)} = 0 && -\frac{\mu b}{3\lambda}w_1^{(2)} + \frac{b}{\lambda}w_2^{(2)} = 0 \\ &&\text{and}& \\ &\frac{\mu a}{3\lambda}w_1^{(1)} + \frac{\mu b}{3\lambda}w_2^{(1)} = 0 && \frac{\mu a}{3\lambda}w_1^{(2)} - \frac{a}{\lambda}w_2^{(2)} = 0 \end{aligned}\right\} \tag{33}$$

Table 1 Convergence criteria

Fitting assumptions	a	b	$(a/\lambda)+(\mu b/3\lambda)$	Convergence criterion
Single-point	$2k\lambda/(1-\mu)$	$-6k\lambda/(1-\mu)$	$2k$	$0<k<1$
Mean-square	$2k/(1+\mu^2)$	$6\mu k\lambda/(1+\mu^2)$	$2k$	$0<k<1$

As in our earlier analytic studies, we assume that the flow is adequately described by the linear, Prandtl-Glauert approximation. At the nth iteration, then, the flowfield will involve both the fundamental (λx) and the harmonic ($3\lambda x$): the perturbation potential will be of the form

$$\phi^{(n)}(x,y) = (A_n e^{\beta\lambda y} + B_n e^{-\beta\lambda y})\cos\lambda x + (a_n e^{3\beta\lambda y} + b_n e^{-3\beta\lambda y})\cos 3\lambda x \qquad (2)$$

The boundary condition enforced by the sinusoidal model is

$$\phi_y(x,0) = V\epsilon\lambda\cos\lambda x \qquad (3)$$

which requires

$$A_n = B_n + \beta^{-1}\epsilon V \qquad (4)$$

and

$$a_n = b_n \qquad (5)$$

At the interface $y=h$, the flow described by Eq. (2) involves the perturbation component

$$\phi_y^{(n)}(x,h) = (A_n e^{\beta\lambda h} - B_n e^{-\beta\lambda h})\beta\lambda\cos\lambda x + a_n(e^{3\beta\lambda h} - e^{-3\beta\lambda h})3\beta\lambda\cos 3\lambda x \qquad (6)$$

The iteration begins by adopting this expression as inner boundary condition for the exterior flowfield; viz.,

$$\varphi^{(n)}(x,y) = C_n e^{-\beta\lambda y}\cos\lambda x + c_n e^{-3\beta\lambda y}\cos 3\lambda x \qquad (7)$$

becomes

$$-(A_n e^{\beta\lambda h} - B_n e^{-\beta\lambda h})e^{-\beta\lambda(y-h)}\cos\lambda x - a_n(e^{3\beta\lambda h} - e^{-3\beta\lambda h})e^{-3\beta\lambda(y-h)}\cos 3\lambda x \qquad (8)$$

The "error signal" $\varphi_x^{(n)}(x,h) - \phi_x^{(n)}(x,h)$ is then found to be

$$\delta^{(n)}u(x,h) \equiv \varphi_x^{(n)}(x,h) - \phi_x^{(n)}(x,h) = 2\lambda A_n e^{\beta\lambda h}\sin\lambda x + 6\lambda a_n e^{3\beta\lambda h}\sin 3\lambda x \qquad (9)$$

We assume that the tunnel operator would like to introduce a correction proportional to this $\delta^{(n)}u(x,h)$, but is limited to functions of the form of $f(x)$ [Eq. (1)] where μ is determined by the equipment and is not under his control. We must also assume how he will choose the constant in Eq. (1) in response to the error signal in Eq. (9).

Two Assumptions Regarding Fitting

Let us investigate the convergence process under two different, rather arbitrarily chosen assumptions.

Single-Point Fitting:

Suppose there is only one measuring instrument in a half-cycle. The operator is then constrained to respond only to the value of $\delta^{(n)}u(x)$ at that value of x. For example, suppose the value at $\lambda x = \pi/2$ is the only datum available. From Eq. (9),

$$\delta^{(n)}u(\pi/2\lambda, h) = 2\lambda A_n e^{\beta\lambda h} - 6\lambda a_n e^{3\beta\lambda h} \qquad (10)$$

Putting the available correction $f^{(n)}(\pi/2\lambda)$ equal to this value, we have

$$(1-\mu)K_n = 2\lambda(A_n e^{\beta\lambda h} - 3a_n e^{3\beta\lambda h}) \qquad (11)$$

and

$$f^{(n)}(x) = \frac{2\lambda}{1-\mu}e^{\beta\lambda h}(A_n - 3a_n e^{2\beta\lambda h})(\sin\lambda x + \mu\sin 3\lambda x) \qquad (12)$$

We therefore manipulate the walls so as to introduce the u increment $kf^{(n)}(x)$ to form the $(n+1)$th approximation.

Mean-Square Fitting:

Alternatively, suppose the wall configuration can be adjusted so as to minimize the difference between the ideal correction, Eq. (9), and the available correction, Eq. (1). This difference is

$$\delta^{(n)}u(x,h) - f^{(n)}(x) = (2\lambda A_n e^{\beta\lambda h} - K_n)\sin\lambda x + (6\lambda a_n e^{3\beta\lambda h} - \mu K_n)\sin 3\lambda x \qquad (13)$$

To minimize this in the mean-square sense, we first calculate the square of Eq. (13) and integrate it over a half-cycle; the result is, except for a multiplicative constant,

$$(2\lambda A_n e^{\beta\lambda h} - K_n)^2 + (6\lambda a_n e^{3\beta\lambda h} - \mu K_n)^2 \qquad (14)$$

Putting the derivative of this expression with respect to K_n equal to zero, we find

$$\frac{K_n}{2\lambda e^{\beta\lambda h}} = \frac{A_n + 3a_n e^{2\lambda\beta h}\mu}{1+\mu^2} \qquad (15)$$

and

$$f^{(n)}(x) = \frac{2\lambda}{1+\mu^2}e^{\beta\lambda h}(A_n + 3a_n e^{2\lambda\beta h}\mu)(\sin\lambda x + \mu\sin 3\lambda x) \qquad (16)$$

In this case we manipulate the walls so as to introduce a u increment equal to k times this function, to form the $(n+1)$th approximation.

Both of the hypotheses regarding fitting of the imperfect correction function have led to formulas of the form†

$$kf^{(n)}(x) = (aA_n e^{\beta\lambda h} + ba_n e^{3\beta\lambda h})(\sin\lambda x + \mu\sin 3\lambda x) \qquad (17)$$

The iterative process consists in introducing this perturbation at the interface, $y=h$; this contributes to the flowfield a potential increment

$$\Delta\phi(x,y) = (\Delta A_n e^{\beta\lambda y} + \Delta B_n e^{-\beta\lambda y})\cos\lambda x + (\Delta a_n e^{3\beta\lambda y} + \Delta b_n e^{-3\beta\lambda y})\cos 3\lambda x \qquad (18)$$

The boundary condition at the model, Eq. (3), cannot be perturbed, and therefore requires

$$\Delta A_n = \Delta B_n \quad \text{and} \quad \Delta a_n = \Delta b_n \qquad (19)$$

Equation (17) constitutes the boundary condition at $y=h$, and gives us

$$\Delta A_n = -\frac{aA_n e^{\beta\lambda h} + ba_n e^{3\beta\lambda h}}{\lambda(e^{\beta\lambda h} + e^{-\beta\lambda h})} \qquad (20)$$

and

$$\Delta a_n = -\frac{\mu}{3\lambda}\,\frac{aA_n e^{\beta\lambda h} + ba_n e^{3\beta\lambda h}}{e^{3\beta\lambda h} + e^{-3\beta\lambda h}} \qquad (21)$$

The perturbed flowfield is described by the potential $\phi^{(n+1)}(x,y)$, which has the same form as Eq. (2) but with A_n,

†Values of a and b for the two fitting assumptions appear in Table 1.

so that

$$K = \begin{bmatrix} -b/a & 1 \\ 1 & \mu/3 \end{bmatrix} \text{ and}$$

$$K^{-1} = -\left(1+\frac{\mu b}{3a}\right)^{-1} \begin{bmatrix} \mu/3 & -1 \\ -1 & -b/a \end{bmatrix} \tag{34}$$

The limiting value of L^m as $m \to \infty$, provided that criterion (30) is satisfied, is clearly

$$L^m \to \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \text{as } m \to \infty \tag{35}$$

The limiting value of A_{n+1} as $n \to \infty$ can now be calculated from Eqs. (26), (31), (34), and (35); it is

$$A_\infty = KL^\infty K^{-1} A_1 = \begin{bmatrix} \dfrac{\mu b}{3a} & -\dfrac{b}{a} \\ -\dfrac{\mu}{3} & 1 \end{bmatrix} \left(1+\frac{\mu b}{3a}\right)^{-1} A_1 \tag{36}$$

where A_1 describes the initial flow situation before the iteration is begun. Thus A_∞ and the final flow depend on the initial flow; this is inevitable, since the amount of the extraneous harmonic introduced must depend on how far the initial flow differs from unconfined flow.‡

Let us now apply this result, Eq. (36), to the two assumed fitting assumptions. The results are given in Table 2.

Since A_∞ is a measure of the residual error in the flow after convergence, it seems clear that for any given $|\mu| < 1$, mean-square fitting leads to a better approximation than single-point fitting at the midpoint of the half-cycle.

Numerical Example

In spite of the fact that the infinite sinusoidal case is an academic one, we think that a numerical example, albeit based on parameters chosen very arbitrarily, may have qualitative value in clarifying the effects of imperfect matching. For this purpose, let us choose the better of the two fitting assumptions, viz., the mean-square fitting assumption. Let us choose $\mu = 1/4$, which means that the tunnel walls can only reproduce a sine curve as a sort of squarewave.

Finally, as has already been pointed out, we have to select some initial conditions. It seems reasonable to say that the uncorrected wall configuration involves no third-harmonic component at all, viz., $a'_1 = 0$; that is, the extraneous harmonic component is only brought into the flow by activation of the imperfect wall-control system.

Thus our numerical example becomes the following:

Initial flow: $A_1 \neq 0, \quad B_1 = -\beta_\epsilon^{-1} V + A_1, \quad a'_1 = 0$

Wall control: $\mu = 1/4$, mean-square fitting

According to Table 2, the converged flow is then described by

$$\left.\begin{aligned} A_\infty &= \frac{\mu^2 A_1}{1+\mu^2} = 0.059 A_1 \\ a'_\infty &= -\frac{\mu}{3}\frac{A_1}{1+\mu^2} = -0.078 A_1 \end{aligned}\right\} \tag{37}$$

‡It might be pointed out that A_2 is identical with A_∞ if $k = ½$. This is called one-step iteration by Lo and Kraft.[4]

Table 2 Final flow

Fitting assumptions	A_∞	a'_∞
Single-point	$(-\mu A_1 + 3a'_1)/(1-\mu)$	$(-\mu/3 A_1 + a'_1)/(1-\mu)$
Mean-square	$(\mu^2 A_1 - 3\mu a'_1)/(1+\mu^2)$	$(-\mu/3 A_1 + a'_1)/(1+\mu^2)$

In Fig. 1 are plotted curves of $\delta^{(1)} u(x,h)$ and $\delta^{(\infty)} u(x,h)$, viz., the initial and ultimate error signals, Eq. (9), in this example. These functions are normalized in Fig. 1 by dividing by $2\lambda e^{\beta\lambda h} A_1$, i.e,

$$\frac{\delta^{(n)} u(x,h)}{2\lambda e^{\beta\lambda h} A_1} = \frac{A_n}{A_1}\sin\lambda x + 3\frac{a'_n}{A_1}\sin 3\lambda x \tag{38}$$

For $n = 1$, a'_n is zero (see above); for $n = \infty$, A_n and a'_n are taken from Eqs. (37). The correction $f^{(1)}/2\lambda e^{\beta\lambda h} A_1$ is also plotted. (The function $f^{(\infty)}/2\lambda e^{\beta\lambda h} A_1$ reaches a maximum value of 0.0004 and cannot be shown on the scale of Fig. 1.)

The question arises: How large are the errors in flow at the model under these conditions, before and after correction? To cast some light on this, the error in $\phi_x^{(n)}(x,0)$ is plotted in Fig. 2. Since the unconfined-flow value is $\lambda\beta^{-1}\epsilon V \sin\lambda x$, the error at the nth iteration is

$$-2A_n\lambda \sin\lambda x - 6a_n\lambda \sin 3\lambda x \tag{39}$$

To plot Fig. 2, the conditions of our numerical example, above, have been chosen, and in addition, since the tunnel/model dimension ratio must be specified to evaluate a_∞, it

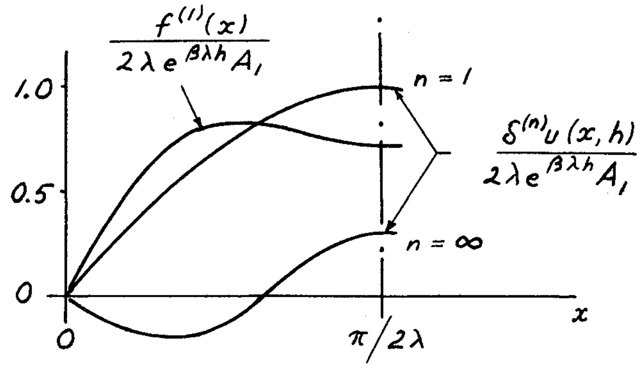

Fig. 1 Error signals $\delta_u^{(n)}(x,h)$ and actual correction $f^{(1)}(x)$. $a'_1 = 0$, $\mu = 1/4$.

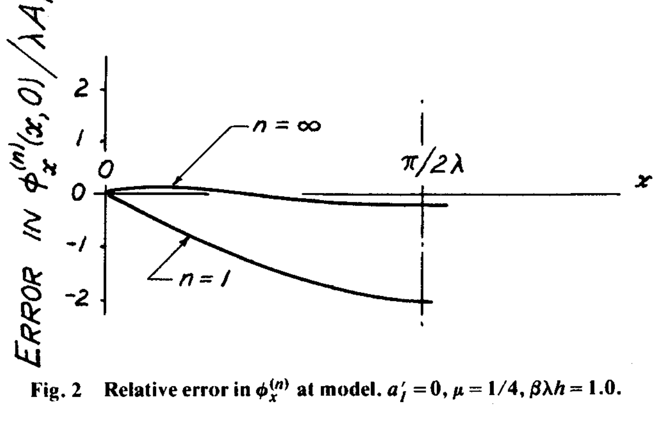

Fig. 2 Relative error in $\phi_x^{(n)}$ at model. $a'_1 = 0$, $\mu = 1/4$, $\beta\lambda h = 1.0$.

has been assumed that $\beta\lambda h=1$. Thus $a_\infty = a'_\infty e^{-2\beta\lambda h} = -0.078\times 0.1353 A_l = -0.01055 A_l$.

In this example, since a_l has been assumed to be zero, i.e., no extraneous harmonic in the uncorrected flow, the error at the model before correction is proportional to A_l. (With a solid wall at $y=h$, $A_l = -\beta^{-l}\epsilon V e^{-2\beta\lambda h} = -0.1353\ \beta^{-l}\epsilon V$ in this example.) After iteration, the maximum error in $\phi_x(x,0)$ is reduced by a factor of $0.1813/2 = 0.09$.

Conclusions

This preliminary, simplified investigation of the effects of imperfect wall control in an adaptable-wall wind tunnel suggests that: 1) the iterative process can still converge but 2) to an imperfect approximation to unconfined flow in which 3) the flow errors at the model may be substantially smaller than before iteration.

Acknowledgments

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